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Galilean covariant models of bosons coupled to a Chern–Simons gauge field

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Abstract
We construct non-relativistic models of complex scalar bosons coupled to Chern–Simons gauge fields by using a Galilean covariant formulation based on the embedding of the $(d, 1)$ Newtonian spacetime into a $(d + 1, 1)$ Minkowski manifold with light-cone coordinates. We also examine various generalizations of the Carroll–Field–Jackiw three-dimensional Chern–Simons term for which the usual Lorentz covariance is broken. Models with cubic Chern–Simons term and non-Abelian gauge fields are briefly discussed. Our main result is the application of this covariant formalism to the investigation of the Aharonov–Bohm effect, for which we retrieve the invariant scattering amplitude up to one-loop.

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1. Introduction

During the last two decades, various non-relativistic systems have been described with the help of a covariant formulation of Galilean field theories in five dimensions [1–8]. This approach consists of building Galilean-covariant actions, as it is usually done with Lorentz covariance, except that we begin with a (4, 1) Minkowski manifold. Non-relativistic theories based on Chern–Simons (CS) gauge fields have also been largely explored. The CS form appeared in mathematics nearly 30 years ago [9], and physicists began to employ it soon thereafter [10]. These theories have been studied mostly in planar physics, but they may be defined also in higher odd-dimensional spacetimes. However, they can be both Lorentz-covariant and quadratic in the gauge field in (2, 1) spacetime only. Some reviews of CS theories are given in [11–13]. In the first investigation of a non-relativistic theory of bosons interacting with Chern–Simons fields, Jackiw and Pi have constructed a theory for quantum system of many point particles, with the CS term tying magnetic flux to the particle fields. Moreover, they have discussed various properties and soliton solutions of their model [14].

Besides, CS theories are used to give a field-theoretical description of the Aharonov–Bohm (AB) effect, which describes the quantum effect of electromagnetic potentials on charged particles; more specifically, the scattering of the particles by a magnetic flux tube [15]. This approach was first proposed in [16] and it was shown in [17] that it is necessary to include a contact interaction among the boson fields in order to ensure the renormalizability in a perturbative field-theoretical treatment of the AB problem. In addition, the non-relativistic limit of the relativistic theory of the AB problem has been considered [18], as well as relativistic corrections [19].

In this paper, we study field models with non-relativistic massive spinless bosons coupled to a CS gauge field, thereby following the same lines as in [3–8], where various applications of a covariant formulation of Galilean field theories are discussed. We express well-known non-relativistic CS Lagrangians in a covariant form, and then propose some generalizations. Our main objective is to retrieve the AB scattering amplitude, at the tree-level and one-loop corrections, with a Galilean-covariant path integral quantization.

The algorithm utilized henceforth in order to implement the Galilei symmetry consists in defining manifestly covariant Lagrangians on a $(d + 1, 1)$ Minkowski manifold, and then reducing them onto the $(d, 1)$ Newtonian spacetime. Such an embedding of the Galilean space into the extended space is based on the fact that the non-trivial central extension of the 11-dimensional Galilei algebra in (3, 1) dimensions is a Lie subalgebra of the 15-dimensional Poincaré algebra in (4, 1) dimensions [20]. Since not all representations of the Galilei group in (3, 1) spacetime can be obtained from representations of the Poincaré group in (4, 1) manifold, this description is therefore not likely to be complete, i.e. there exist Galilean invariant theories which cannot be described by this reduction. The determination of these representations remains an open question. For instance, the reduction procedure in (2, 1) spacetime describes one central charge only, whereas the Galilei Lie algebra then admits *two* central charges.

Following [3–8], let us define a *Galilei vector* $X = (X^1, \ldots, X^d, X^4, X^5)$ as transforming under a Galilean boost with relative velocity $\mathbf{V} = (V_1, \ldots, V_d)$ according to

$$\mathbf{X}' = \mathbf{X} - \mathbf{V}X^4, \qquad X^{4\prime} = X^4, \qquad X^{5\prime} = X^5 - \mathbf{V}\cdot\mathbf{X} + \tfrac{1}{2}\mathbf{V}^2 X^4.$$

The new coordinate X^5 may be associated with the quasi-invariance of non-relativistic Lagrangians or, equivalently, with the phase that must be included into the quantum wavefunction, so that the Schrödinger equation be invariant under the Galilei transformations [21]. The *Galilean scalar product* of two vectors is invariant under these transformations, with the *Galilean metric* given by

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} \mathbf{1}_{d\times d} & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}.$$

For planar physics, discussed hereafter, we take $d = 2$, so that we begin with a (3, 1) spacetime. The reduction approach to Galilean invariance has been investigated from different points of view by many authors [22]. Since the Poincaré Lie algebra of the extended manifold includes the Galilei as well as the Poincaré Lie algebras, this approach provides a unifying geometrical view for both non-relativistic and relativistic physics. In particular, it has allowed us to unify various d-brane fluid models: the Nambu–Goto relativistic d-brane moving in $(d + 1, 1)$ Minkowski space reduces to the *non-relativistic* Chaplygin gas model, on one hand, and to the *relativistic* Born–Infeld model, on the other hand [23, 24].

Let us denote the five-coordinate vector as

$$x^\mu = (\mathbf{x}, t, s). \tag{1}$$

With the usual relations, $E = \mathrm{i}\partial_t$, as well as $m = \mathrm{i}\partial_s$, we may write the five-momentum as

$$p_\mu \equiv -\mathrm{i}\partial_\mu = (-\mathrm{i}\nabla, -\mathrm{i}\partial_t, -\mathrm{i}\partial_s) = (\mathbf{p}, -E, -m). \tag{2}$$

This provides an interpretation of the fifth coordinate: it is conjugate to the mass, in a way similar to the energy conjugate of time, and linear momentum conjugate of position.

If we act on a wavefunction with the invariants $P^\mu P_\mu$ and P_5 of the Poincaré algebra in (4, 1) dimensions, then we find, using equation (2):

$$\partial_\mu\partial^\mu\tilde{\Phi}(x) = k^2\tilde{\Phi}(x), \qquad \partial_5\tilde{\Phi}(x) = -\mathrm{i}m\tilde{\Phi}(x),$$

where x is a five-dimensional vector. The first equation, together with equation (2), leads to the dispersion relation $\mathbf{p}^2 - 2mE = -k^2$. (If we define $k^2 = 2m^2$, then we find $E = \frac{\mathbf{p}^2}{2m} + m$.) From the second equation above, we find that the fifth coordinate can be factored out of the wavefunction as follows:

$$\tilde{\Phi}(x) = \mathrm{e}^{-\mathrm{i}mx^5}\Phi(\mathbf{x}, x^4) = \mathrm{e}^{-\mathrm{i}ms}\Phi(\mathbf{x}, t). \tag{3}$$

It follows therefrom that massless fields are independent of x^5. With equation (1), the first equation gives the Schrödinger equation for a free massive spinless particle:

$$\mathrm{i}\partial_t\Phi(\mathbf{x}, t) = -\frac{1}{2m}\nabla^2\Phi(\mathbf{x}, t),$$

where we have absorbed the constant k within the energy operator. The extra coordinate x^5, or s, is a real number, and we will interpret any integral over it as

$$\int \mathrm{d}x^5 \to \lim_{l\to\infty}\frac{1}{2l}\int_{-l}^{l}\mathrm{d}s.$$

Therefore, an integral over s will be reduced to the usual integral over (3, 1) spacetime if the integrand is independent of s. This will prevent us from carrying along the factor l.

Our paper is organized as follows. In section 2, we write the general Maxwell–Chern–Simons–Higgs Lagrangian, describing the interaction between a complex scalar boson field and a CS gauge field, and then we discuss various field definitions in terms of x^5. We also propose further generalizations. In section 3, we employ the Galilean-covariant path-integral quantization method [3] to study the Aharonov–Bohm problem. The scattering amplitude is computed up to one-loop. Finally, the concluding remarks are in section 4.

2. Galilei-covariant Lagrangians with Chern–Simons gauge field

In this section, we study the coupling of a spinless scalar field to a Maxwell–Chern–Simons gauge field in the plane [25, 26]. We also propose generalizations such as CS term of higher degree and number of dimensions as well as non-Abelian CS models. Note that henceforth, whenever we consider lower dimensions, we shall denote time and s by x^4 and x^5, respectively, and we will rather delete the coordinate x^2 or x^3, accordingly. Also, throughout the paper, fields or Lagrangians denoted with the tilde mark are defined on the extended space, whereas symbols without the tilde denote fields for which the reduction has been performed. Greek indices label coordinates in extended spacetime, lowercase latin indices a, b, etc denote spatial coordinates and we use uppercase latin indices A, B, etc for field components, gauge indices, etc.

Let us define a model on a (3, 1) Minkowski manifold with coordinates denoted by (x^1, x^2, x^4, x^5), and we reduce it to a (2, 1) Newtonian spacetime with coordinates (x^1, x^2, x^4).

Consider a spinless scalar field interacting with an Abelian CS gauge field A described by the Maxwell–Chern–Simons–Higgs Lagrangian:

$$\tilde{\mathcal{L}}_{\mathrm{MCSH}} = \beta(\tilde{D}_\mu\tilde{\Phi})^*(\tilde{D}^\mu\tilde{\Phi}) + \frac{\kappa}{2}\epsilon^{\mu\nu\rho}\tilde{A}_\mu\partial_\nu\tilde{A}_\rho + \frac{\gamma}{4}\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu} - k^2|\tilde{\Phi}|^2 - V(|\tilde{\Phi}|), \tag{4}$$

where β is a constant, $\tilde{F}_{\mu\nu} \equiv \partial_\mu\tilde{A}_\nu - \partial_\nu\tilde{A}_\mu$ and $\tilde{D}_\mu \equiv \partial_\mu - \mathrm{i}e\tilde{A}_\mu$. Later, it will be an easy matter to set equal to zero any constant, should we wish to neglect a particular term. The situation in which e, κ and γ are equal to zero has been quantized in [3] (therein, the fourth term of equation (4) reads $+\frac{k^2}{2m}|\tilde{\Phi}|^2$). In the relativistic case, the Maxwell term remedies the absence of transverse components in CS fields which, in turn, leads to an inconsistency with the path integral over the gauge field. In addition to restoring the convergence of the path integral over the gauge fields, the Maxwell term provides a gauge-invariant regularization and renormalization of the theory [27]. However, we will not include it in our analysis of the AB effect, discussed in section 3.

The totally antisymmetrical tensor in the CS term should be understood as $\epsilon^{\mu\nu\rho} = \epsilon^{5\mu\nu\rho}$, i.e. the indices within $\epsilon^{\mu\nu\rho}$ run over 1, 2, 4 only. More on this will be given shortly. The variations of the corresponding action with respect to $\tilde{\Phi}^*$ and $\tilde{A}_\mu$ give

$$\beta\tilde{D}_\mu\tilde{D}^\mu\tilde{\Phi} = -\frac{\delta V}{\delta\tilde{\Phi}^*} - k^2\tilde{\Phi}, \tag{5}$$

and

$$\frac{\kappa}{2}\epsilon^{\mu\alpha\beta}\tilde{F}_{\alpha\beta} = -e\tilde{j}^\mu + \gamma\partial_\alpha\tilde{F}^{\alpha\mu}, \tag{6}$$

respectively, where we have defined the five-current

$$\tilde{j}^\mu = \mathrm{i}\beta(\tilde{\Phi}^*(\tilde{D}^\mu\tilde{\Phi}) - (\tilde{D}^\mu\tilde{\Phi})^*\tilde{\Phi}). \tag{7}$$

Let us choose the potential in equation (4) as follows:

$$V(|\tilde{\Phi}|) = \frac{\lambda}{4}|\tilde{\Phi}|^4. \tag{8}$$

Then, equation (5) becomes

$$\beta\tilde{D}_\mu\tilde{D}^\mu\tilde{\Phi} = -\frac{\lambda}{2}|\tilde{\Phi}|^2\tilde{\Phi} - k^2\tilde{\Phi}. \tag{9}$$

The appearance of this potential in the second-quantized Hamiltonian leads to delta-function interactions between particles in the corresponding first-quantized theory. Dimock has shown, for one spatial dimension, that the delta-function potential of quantum mechanics is the non-relativistic limit of the relativistic $\lambda\Phi^4$ theory [28]. The contact interaction of anyons has been considered in [29]. Such models lead to ultraviolet divergences which require regularization and renormalization [30]. Also, note that at the so-called *self-dual* coupling point $\lambda = \pm 1/|\kappa|$, the renormalized scattering amplitude becomes scale independent [17].

As mentioned above, the CS term of equation (4) vanishes at once if one defines the embedding as in equation (1) and if the five-potential $\tilde{A}_\mu$ does not depend on s. This is the case, unless the indices take on the values 1, 2, 4, which is achieved by modifying the CS term as follows [31]:

$$\tfrac{1}{2}\kappa\epsilon^{\mu\nu\alpha\beta}v_\mu\tilde{A}_\nu\partial_\alpha\tilde{A}_\beta,$$

where we take $v_\mu = (\mathbf{0}, 0, 1)$. In relativistic theories, this term is gauge invariant, but, unlike the usual CS term, it is not Lorentz invariant [31]. Henceforth, this choice of v_μ simply amounts to replacing $\epsilon^{\alpha\mu\nu\rho}$ with $\epsilon^{5\mu\nu\rho}$, so that we can still use the Lagrangian of equation (4), except that the indices of $\epsilon^{\mu\nu\rho}$ run over 1, 2, 4, unlike the other terms of the Lagrangian, where the indices are 1, 2, 4, 5. Henceforth, we utilize the convention $\epsilon^{1245} = 1$, so that, for instance, $\epsilon^{124} = -1$.

In a recent publication [8], it was observed that the 'magnetic' limit of Maxwell equations investigated by Le Bellac and Lévy–Leblond [32] may be obtained by substituting $\tilde{A}_\mu = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_4, \tilde{A}_5) = (\mathbf{A}(\mathbf{x}, x^4), -\phi(\mathbf{x}, x^4), 0)$ into the covariant form of the equations of motion. The other possibility, called 'electric' limit, is obtained by $\tilde{A}_\mu = (\mathbf{A}(\mathbf{x}, x^4), 0, -\phi(\mathbf{x}, x^4))$. However, we have observed recently that this choice cannot lead to the Lagrangians which correspond to the respective Galilean limits [5]. The reason is that by putting gauge field components equal to zero before we compute the Euler–Lagrange equations, these missing components do not allow us to recover the full set of equations. The Galilean electric and magnetic Lagrangians both involve auxiliary fields, which may be set equal to zero only after the field equations have been obtained. The proper way to construct these Lagrangians, within the present formalism, is to define

$$\tilde{A}_\mu = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_4, \tilde{A}_5) = (\mathbf{A}(\mathbf{x}, t), -\phi_M(\mathbf{x}, t), -\phi_E(\mathbf{x}, t)). \tag{10}$$

The two components ϕ_M and ϕ_E should not be understood as two simultaneously physical scalar potentials. The Lagrangian which corresponds to the magnetic limit is retrieved by considering ϕ_E as an auxiliary field, set equal to zero in the equations of motion, whereas ϕ_M is the physical scalar potential field. With equation (10), we get the following components of the electromagnetic field strength tensor: $F_{4a} = \partial_a\phi_M + \partial_t A_a$, $F_{5a} = \partial_a\phi_E$ and $F_{54} = \partial_t\phi_E$. Note also that the Galilean form of the Lorentz gauge condition reads

$$\partial_\mu A^\mu = \nabla\cdot\mathbf{A} + \partial_t\phi_E + \partial_s\phi_M = \nabla\cdot\mathbf{A} + \partial_t\phi_E,$$

since the fields ϕ are massless, i.e. $\partial_s\phi = 0$. This remains as it is for the electric limit (for which $\phi_M = 0$) but it reduces to the Coulomb gauge condition in the magnetic limit (for which $\phi_E = 0$) [8]. More about the gauge conditions and the Galilean electromagnetism may be found in [33].

If we utilize equations (1), (3), (8) and (10), then the Lagrangian of equation (4) becomes

$$\begin{aligned}\mathcal{L} = {} & \beta[(\mathbf{D}\Phi)^*\cdot\mathbf{D}\Phi + \mathrm{i}(m - e\phi_E)(\Phi(D_t\Phi)^* - \Phi^* D_t\Phi)] + \frac{\kappa}{2}[\mathbf{A}\times\partial_t\mathbf{A} + \mathbf{A}\times\nabla\phi_M \\ & + \phi_M B] + \frac{\gamma}{2}[B^2 - 2(\nabla\phi_M + \partial_t\mathbf{A})\cdot\nabla\phi_E - (\partial_t\phi_E)^2] - k^2|\Phi|^2 - \frac{\lambda}{4}\Phi^4,\end{aligned} \tag{11}$$

where Φ is a function of $\mathbf{x}$ and t only and

$$B \equiv F_{xy} = \partial_x A_y - \partial_y A_x. \tag{12}$$

Since $\mathbf{x}$ is a two-dimensional vector, the magnetic field $\nabla\times\mathbf{A}$ is not a vector but a pseudo-scalar. Note that the term $\mathbf{A}\times\nabla\phi$ is absent in [17].

From equation (9), with similar definitions, we find:

$$\begin{aligned}\mathrm{i}\left(1 - \frac{e}{m}\phi_E\right)\partial_t\Phi(\mathbf{x}, t) = {} & \left[-\frac{1}{2m}\mathbf{D}^2 + e\left(1 - \frac{e}{m}\phi_E\right)\phi_M \right. \\ & \left. + \mathrm{i}\frac{e}{m}\partial_t\phi_E - \frac{k^2}{2m\beta} - \frac{\lambda}{4m\beta}|\Phi(\mathbf{x}, t)|^2\right]\Phi(\mathbf{x}, t),\end{aligned}$$

where $\mathbf{D} = \nabla - \mathrm{i}e\mathbf{A}$ and $D_t = \partial_t + \mathrm{i}e\phi_M$. With the overall constant β defined as

$$\beta = -\frac{1}{2m}, \tag{13}$$

and by setting $\phi_E = 0$, so that we work in the magnetic limit, then we find the nonlinear planar Schrödinger equation with coupling to a gauge field:

$$\mathrm{i}\partial_t\Phi(\mathbf{x}, t) = \left(-\frac{1}{2m}\mathbf{D}^2 + e\phi_M + k^2 + \frac{\lambda}{2}|\Phi(\mathbf{x}, t)|^2\right)\Phi(\mathbf{x}, t).$$

From equations (7), (10) and (13), we get

$$
\begin{aligned}
j^a &= \frac{1}{2m\mathrm{i}}(\tilde{\Phi}^*(D_a\tilde{\Phi}) - \tilde{\Phi}(D_a\tilde{\Phi})^*),\\
j^4 &= -j_5 = \frac{m - e\phi_E}{m}|\Phi(\mathbf{x},t)|^2 \to |\Phi(\mathbf{x},t)|^2,\\
j^5 &= -j_4 = \frac{1}{2m\mathrm{i}}((\partial_t\Phi^*)\Phi - \Phi^*\partial_t\Phi - 2\mathrm{i}e\phi_M|\Phi|^2).
\end{aligned}
\tag{14}
$$

Now let us turn to the components of equation (6). For $\mu = 4$, with equations (3) and (10), we find

$$
B = -\frac{e}{\kappa}\left(\frac{m - e\phi_E(\mathbf{x},t)}{m}\right)|\Phi(\mathbf{x},t)|^2 + \gamma\nabla^2\phi_E(\mathbf{x},t),
$$

which, when $\phi_E = 0$, becomes

$$
B = -\frac{e}{\kappa}|\Phi(\mathbf{x},t)|^2. \tag{15}
$$

For $\mu = 1, 2$, equation (6) leads to

$$
\epsilon^{ab}\left(\partial_t A_b + \partial_b\phi_M + \frac{\gamma}{\kappa}\partial_b B\right) = -\frac{1}{\kappa}(ej^a + \gamma\partial_t\partial_a\phi_E), \tag{16}
$$

where $a, b = 1, 2$, with j^a in equation (14) being the two components of the probability current, and the totally antisymmetric tensor such that $\epsilon^{12} = +1$. The magnetic induction B is given by equation (12). Equations (15) and (16) characterize the CS gauge field. The first expression exhibits the linearity between the density of charge ρ and the magnetic field $\nabla\times\mathbf{A}$, and the second equation ensures the conservation of the particles flux during time evolution. Thus, the manifestly Galilean-covariant form of these equations is given by equations (5) and (6), with the potential as in equation (8) and $\gamma = 0$. Similarly, the Lagrangian density considered at the beginning of this section is the Galilei-covariant form of the one used in [17]. In section 3, it will enable us to quantize this model with a covariant approach.

Before we apply our approach to the AB effect, let us briefly discuss some generalizations. The 'genuine' Chern–Simons theories are well defined in odd-dimensional spacetimes only. They do not violate Lorentz symmetry, unlike the models investigated in [31]. Therefore, the Galilean cubic CS interaction is defined in a five-dimensional manifold, with four spatial dimensions. Then, by performing the embedding of the Newtonian spacetime into this five-dimensional manifold, we end up with a non-relativistic model in (3,1) spacetime. Let us define the Lagrangian:

$$
\tilde{\mathcal{L}}_{3\mathrm{MCSH}} = -\frac{1}{2m}(\tilde{D}_\mu\tilde{\Phi})^*(\tilde{D}^\mu\tilde{\Phi}) + \frac{\kappa}{2}\epsilon^{\alpha\mu\nu\sigma\tau}\tilde{A}_\alpha\partial_\mu\tilde{A}_\nu\partial_\sigma\tilde{A}_\tau + \frac{\gamma}{4}\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu} - k^2|\Phi|^2 - V(|\tilde{\Phi}|),
$$

where we use equation (13). From the embedding given in equation (1) with the fields defined as in equations (3) and (10), this Lagrangian reduces to

$$
\begin{aligned}
\mathcal{L}_{3\mathrm{MCSH}} = {}& -\frac{1}{2m}|\mathbf{D}\Phi|^2 - \frac{\mathrm{i}}{2m}(m - e\phi_E)((D_t\Phi)^*\Phi - \Phi^*D_t\Phi) + \kappa\phi_E\mathbf{B}\cdot(\partial_t\mathbf{A} + \nabla\phi_M)\\
& - \kappa\phi_M\mathbf{B}\cdot\nabla\phi_E - \kappa\mathbf{A}\cdot[\mathbf{B}\partial_t\phi_E + (\partial_t\mathbf{A} + \nabla\phi_M)\times\nabla\phi_E] + \frac{\gamma}{2}\mathbf{B}^2\\
& - \gamma(\partial_t\mathbf{A} + \nabla\phi_M)\cdot\nabla\phi_E - \frac{\gamma}{2}(\partial_t\phi_E)^2 - k^2|\Phi|^2 - V(|\Phi|).
\end{aligned}
\tag{17}
$$

The boldface font now denotes three-dimensional vectors. If we take $\phi_E = 0$, then the CS term disappears completely from the Lagrangian.

The Euler–Lagrange equations with respect to the field A_α give

$$
\tfrac{3}{2}\epsilon^{\alpha\mu\nu\sigma\tau}\partial_\mu\tilde{A}_\nu\partial_\sigma\tilde{A}_\tau = -e\tilde{j}^\alpha + \gamma\partial_\mu\tilde{F}^{\mu\alpha}, \tag{18}
$$

where $\tilde{j}^{\alpha}$ is given in equation (7). For $\alpha = 1, 2, 3$, this equation becomes, by using equations (1), (3) and (10):

$$\gamma\nabla\times\mathbf{B} + \gamma\partial_t\nabla\phi_E - 3\mathbf{B}\partial_t\phi_E + 3\nabla\phi_E\times(\partial_t\mathbf{A} + \nabla\phi_M) = -e\mathbf{j}.$$

Obviously, this may be obtained by variations of equation (17) with respect to the components of $\mathbf{A}$. If we consider the magnetic limit, for which $\phi_E = 0$, this reduces further:

$$\gamma\nabla\times\mathbf{B} = -e\mathbf{j}.$$

Let us take equation (18) with $\alpha = 4$ or, equivalently, we vary equation (17) with respect to the field ϕ_M, then we obtain

$$3\mathbf{B}\cdot\nabla\phi_E + \gamma\nabla^2\phi_E = ej^4 = e\frac{m - e\phi_E}{m}|\Phi(\mathbf{x}, t)|^2.$$

It becomes $|\Phi(\mathbf{x}, t)|^2 = 0$ in the magnetic limit, for which $\phi_E = 0$. From equation (18) with $\alpha = 5$, we find

$$(3\mathbf{B} + \gamma\nabla)\cdot(\partial_t\mathbf{A} + \nabla\phi_M) - \gamma\partial_{tt}\phi_E = ej^5 = \frac{e}{2m\mathrm{i}}((\partial_t\Phi^*)\Phi - \Phi^*\partial_t\Phi - 2\mathrm{i}e\phi_M|\Phi|^2).$$

For $\phi_E = 0$, this becomes

$$(3\mathbf{B} + \gamma\nabla)\cdot(\partial_t\mathbf{A} + \nabla\phi_M) = \frac{e}{2m\mathrm{i}}((\partial_t\Phi^*)\Phi - \Phi^*\partial_t\Phi - 2\mathrm{i}e\phi_M|\Phi|^2).$$

Let us conclude this discussion with non-Abelian CS models. Relativistic models have been considered in [34], and the corresponding non-relativistic theory has been investigated in [35]. The perturbative treatment of the non-Abelian AB effect to one-loop has been studied in [36].

For a non-Abelian gauge group, generated by a Lie algebra with basis elements $\{t^1, t^2, \ldots, t^n\}$ satisfying the commutation relations:

$$[t^A, t^B] = \mathrm{i}f^{ABC}t^C,$$

the Lagrangian in equation (4) may be generalized to

$$\tilde{\mathcal{L}}_{\mathrm{NAMCSH}} = -\frac{1}{2m}(\tilde{D}_\mu\tilde{\Phi})^*\left(\tilde{D}^\mu\tilde{\Phi}\right) + \frac{\kappa}{2}\epsilon^{\mu\nu\rho}\,\mathrm{Tr}\left(\tilde{A}_\mu\partial_\nu\tilde{A}_\rho - \frac{2}{3}ig\tilde{A}_\mu\tilde{A}_\nu\tilde{A}_\rho\right) + \frac{\gamma}{4}\,\mathrm{Tr}(\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu}) - V(|\tilde{\Phi}|), \tag{19}$$

where the trace is over the gauge group index. We have chosen $\beta = -1/2m$, as in equation (13). The relation between the field strength tensor $\tilde{F}_{\mu\nu} = \frac{\mathrm{i}}{g}[\tilde{D}_\mu, \tilde{D}_\nu]$ and the covariant derivative $\tilde{D}_\mu = \partial_\mu - \mathrm{i}g\tilde{A}^a_\mu t^a$ provides the usual expression:

$$\tilde{F}^c_{\mu\nu} = \partial_\mu\tilde{A}^c_\nu - \partial_\nu\tilde{A}^c_\mu + gf^{abc}\tilde{A}^a_\mu\tilde{A}^b_\nu.$$

For the gauge algebra su(2), generated by the Pauli matrices, the CS term becomes:

$$\epsilon^{\mu\nu\rho}\frac{\kappa}{2}\,\mathrm{Tr}\left(\tilde{A}^A_\mu t^A\partial_\nu\tilde{A}^B_\rho t^B - \frac{2}{3}\mathrm{i}\tilde{A}^A_\mu t^A\tilde{A}^B_\nu t^B\tilde{A}^C_\rho t^C\right) = \epsilon^{\mu\nu\rho}\kappa\left(\delta_{AB}\tilde{A}^A_\mu\partial_\nu\tilde{A}^B_\rho + \frac{2}{3}g\epsilon_{ABC}\tilde{A}^A_\mu\tilde{A}^B_\nu\tilde{A}^C_\rho\right),$$

where the capital latin letters denote su(2) indices. With the embedding given in equations (1), (3) and (10), the CS term reduces to

$$\kappa\delta_{AB}\left[\left(A^A_y\partial_x\phi^B - A^A_x\partial_y\phi^B\right) - \phi^A B^B + \left(A^A_y\partial_t A^B_x - A^A_x\partial_t A^B_y\right)\right] - \tfrac{2}{3}\kappa g\epsilon_{ABC}\left[A^A_x A^B_y\phi^C + A^A_y\phi^B A^C_x + \phi^B A^B_x A^C_y - A^A_y A^B_x\phi^C - A^A_x\phi^C A^B_x - \phi^A A^B_y A^C_x\right],$$

where $B^C \equiv \partial_x A^C_y - \partial_y A^C_x$.

3. Covariant formulation of the Aharonov–Bohm problem

As an application of the models discussed above, we now study the AB effect for bosons in a field-theoretical approach, as done in [17]. However, we use a covariant path-integral approach, as was done in [3], with the Lagrangian density of equation (4), where some terms are set equal to zero. The Galilean-covariant generating functional has the usual form:

$$Z[\tilde{J}(x), \tilde{J}^*(x), \tilde{j}_\mu(x)] = \frac{\int \mathcal{D}\tilde{\Phi}\mathcal{D}\tilde{\Phi}^*\mathcal{D}A_\mu\, \mathrm{e}^{\mathrm{i}I - \mathrm{i}\int \mathrm{d}^4x(\tilde{J}(x)\tilde{\Phi}^*(x) + \tilde{J}^*(x)\tilde{\Phi}(x) - \tilde{j}_\mu(x)\tilde{A}^\mu(x))}}{\int \mathcal{D}\tilde{\Phi}\mathcal{D}\tilde{\Phi}^*\mathcal{D}A_\mu\, \mathrm{e}^{\mathrm{i}I}}. \tag{20}$$

where $\mathrm{d}^4x = \mathrm{d}x^1\,\mathrm{d}x^2\,\mathrm{d}x^4\,\mathrm{d}x^5$. The action I is usually split as follows:

$$I = I_\Phi + I_{\mathrm{CS}} + I_{\mathrm{GF}} + I_{\mathrm{Maxw}},$$

with the $\tilde{\Phi}$ contribution:

$$I_\Phi = \int \mathrm{d}^4x \left[-\frac{1}{2m}(\tilde{D}_\mu\tilde{\Phi})^*\tilde{D}^\mu\tilde{\Phi} - k^2|\tilde{\Phi}|^2 - V(|\tilde{\Phi}|) \right],$$

(using equation (13)) the CS term:

$$I_{\mathrm{CS}} = \int \mathrm{d}^4x\, \frac{\kappa}{2}\epsilon^{\mu\nu\rho}\tilde{A}_\mu\partial_\nu\tilde{A}_\rho,$$

the gauge fixing term:

$$I_{\mathrm{GF}} = \int \mathrm{d}^4x\, \frac{1}{\xi}(\partial_\mu\tilde{A}^\mu)^2,$$

and the regularizing Maxwell term:

$$I_{\mathrm{Maxw}} = \int \mathrm{d}^4x\, \frac{\gamma}{4}\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu}.$$

It is known that the covariant formulation of the path-integral quantization of the non-relativistic scalar field yields the following bosonic propagator in the extended momentum space [3]:

$$\overline{\Delta}_F(p) = -\frac{2p^4}{(p_\mu p^\mu - k^2 - \mathrm{i}\varepsilon)},$$

with the following Fourier transform,

$$\Delta_F(x-y) = \frac{1}{(2\pi)^4}\int \mathrm{d}^4p\, \overline{\Delta}_F(p)\,\mathrm{e}^{\mathrm{i}p\cdot(x-y)}[2\pi\delta(p^4 - m)]. \tag{21}$$

The CS gauge field is not dynamical, as shown by equations (15) and (16). Therefore, in a quantum field theory, it is treated only as a field which generates internal propagations, that is to say, virtual particles that propagate instantaneously in time. The Fourier transform of the gauge propagator is

$$\overline{D}_{\mu\nu}(w-z) = \frac{1}{(2\pi)^4}\int \mathrm{d}^4p\,\overline{D}_{\mu\nu}(p)\,\mathrm{e}^{\mathrm{i}p\cdot(w-z)}\delta(p^4). \tag{22}$$

Hereafter we take $\gamma = 0$, so that we have

$$\overline{D}_{\mu\nu}(\hat{p}) = \frac{\mathrm{i}\epsilon_{\mu\nu\rho}\hat{p}^\rho}{\kappa\hat{p}^2} + \frac{\xi\hat{p}_\mu\hat{p}_\nu}{p^4}.$$

The symbol $\hat{p}$ denotes the momentum vector given in equation (2), but with $m = 0$. Note that this choice generates directly the ordinary gauge propagator, since $\hat{p}^2 = \mathbf{p}^2$. In the

Landau gauge, for which $\xi = 0$, the gauge field propagator has the following non-vanishing components (remembering that $\epsilon_{\mu\nu\rho} = \epsilon_{5\mu\nu\rho}$),

$$\overline{D}_{i4}(\hat{p}) = \frac{\mathrm{i}\epsilon_{ij}\hat{p}^j}{\kappa\hat{p}^2}.$$

Therefore, the path-integral quantization formalism developed in [3] can also be employed in the study of the present case. Also, it is worth mentioning that the presence of the delta functions in equations (21) and (22) makes this approach compatible with the fact that the momentum component P^4, equivalent to the mass, is Galilean invariant. Therefore, it plays the role of the mass of the field quanta, in a quantized formalism. The argument of the delta function in (22) is due to the fact that the CS gauge field is massless.

Now let us determine the scattering amplitude of the system described by equation (20). We denote by the ket $|p_1, p_2\rangle$ the initial state of two non-relativistic particles described by the scalar field $\Phi(x)$, and $|p_1', p_2'\rangle$ represents the final state. Then the scattering amplitude is defined by

$$S_{fi} = \langle p_1', p_2'|S|p_1, p_2\rangle,$$

with the operator S given by

$$S =: \exp\left\{\int \mathrm{d}^4x\left[\hat{\Phi}_{\mathrm{in}}(x)O_x\frac{\delta}{\delta\tilde{J}(x)} + \hat{\Phi}_{\mathrm{in}}^{\dagger}(x)O_x\frac{\delta}{\delta\tilde{J}^*(x)}\right]\right\} : Z[\tilde{J}, \tilde{J}^*, \tilde{j}_\mu]|_{J=J^*=\tilde{j}_\mu=0}, \tag{23}$$

where the operator $\hat{\Phi}_{\mathrm{in}}$ is the field in the initial configuration, $O_x = -\beta\partial_\mu\partial^\mu$, and :: denotes the normal ordering (details are given in [3]). The expansion of the exponential in equation (23) yields several terms, but we only show the physically interesting one:

$$S_{fi}^{(4)} = \int \mathrm{d}^4x_1\,\mathrm{d}^4x_2\,\mathrm{d}^4x_3\,\mathrm{d}^4x_4\,\mathrm{e}^{\mathrm{i}(p_1\cdot x_1+p_2\cdot x_2-p_1'\cdot x_3-p_2'\cdot x_4)}O_{x_1}O_{x_2}O_{x_3}O_{x_4}G(x_1, x_2, x_3, x_4).$$

This is the reduction formula for the four-point Green function, given by

$$G(x_1, x_2, x_3, x_4) = \left.\frac{\delta^4 Z[\tilde{J}, \tilde{J}^*, \tilde{j}_\mu]}{\delta\tilde{J}^*(x_1)\delta\tilde{J}^*(x_2)\delta\tilde{J}(x_3)\delta\tilde{J}(x_4)}\right|_{\tilde{J}=\tilde{J}^*=\tilde{j}_\mu=0}.$$

Since the CS particles exist only in the intermediate processes, it justifies our interest in the scattering processes involving only matter field quanta. Therefore, gauge propagators appear as internal lines of Feynman diagrams.

For convenience, let us define the invariant amplitude $A(p_1, p_2, p_1', p_2')$, related to the scattering amplitude S by

$$\begin{aligned}&\langle p_1', p_2'|S - 1|p_1, p_2\rangle\\ &\quad = -\mathrm{i}(2\pi)^4\tilde{\delta}^4(p_1 + p_2 - p_1' - p_2')M\left(p_1^4, p_2^4, p_1'^4, p_2'^4\right)A(p_1, p_2, p_1', p_2').\end{aligned}$$

The factor $M\left(p_1^4, p_2^4, p_1'^4, p_2'^4\right)$ carries all the delta functions which involve internal mass conservation in the diagrams, that appear because of the definition of the gauge and particle propagators given by equations (21) and (22).

By following standard methods, we write the generating functional as

$$Z[\tilde{J}(x), \tilde{J}^*(x), \tilde{j}_\mu(x)] = N\exp\left\{-\mathrm{i}\int \mathrm{d}^4x\,\mathcal{L}_{\mathrm{int}}\left[\frac{1}{\mathrm{i}}\frac{\delta}{\delta\tilde{J}^*}, \frac{1}{\mathrm{i}}\frac{\delta}{\delta\tilde{J}}, \frac{1}{\mathrm{i}}\frac{\delta}{\delta\tilde{j}_\mu}\right]\right\}Z_0[\tilde{J}, \tilde{J}^*, \tilde{j}_\mu], \tag{24}$$

where $N = Z[0, 0, 0]^{-1}$. The term $\mathcal{L}_{\mathrm{int}}$ comes from the interacting part of the action I in equation (20):

$$\mathcal{L}_{\mathrm{int}} = \mathrm{i}e\beta A^\mu[\tilde{\Phi}\partial_\mu\tilde{\Phi}^* - \tilde{\Phi}^*\partial_\mu\tilde{\Phi}] - \beta e^2 A^\mu A_\mu\tilde{\Phi}^*\tilde{\Phi} - \frac{\lambda}{4}|\tilde{\Phi}|^4. \tag{25}$$

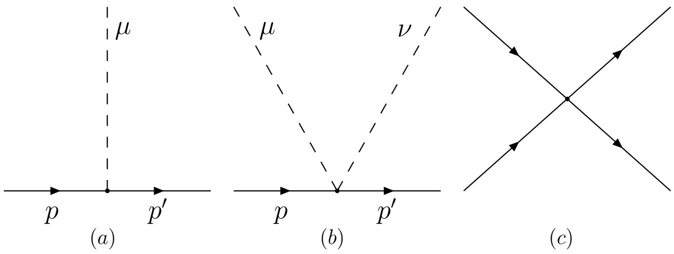

Figure 1. Vertex diagrams for the theory defined in equation (26).

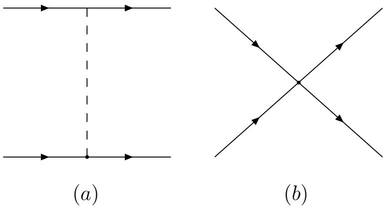

Figure 2. Two-particle scattering at the tree-level.

Note that in equation (24), $\mathcal{L}_{\text{int}}$ appears in the functional representation. The other factor $Z_0[\tilde{J}, \tilde{J}^*, \tilde{j}_\mu]$ can be cast in a convenient form:

$$Z_0[\tilde{J}, \tilde{J}^*, \tilde{j}_\mu] = Z_0\Bigg[0, 0, 0]\exp\left\{-\mathrm{i}\int \mathrm{d}^4x\,\mathrm{d}^4y[\tilde{J}^*(x)\Delta_F(x-y)\tilde{J}(y)\right.$$
$$\left. -\frac{1}{2}\tilde{j}^\mu(x)\overline{D}_{\mu\nu}(x-y)\tilde{j}^\nu(y)\Bigg]\right\}.$$

Finally, the factor $Z_0[0, 0, 0]$ contains the free part of the action I in (20).

We note that the following covariant interaction vertices emerge from the interaction terms in equation (25), expressed as

$$\Gamma^\mu(p+p') = \mathrm{i}\beta e(p^\mu + p'^\mu), \qquad \Gamma^{\mu\nu} = 2\mathrm{i}\beta e^2 g^{\mu\nu}, \qquad \Gamma_\lambda = \mathrm{i}\lambda, \tag{26}$$

as displayed in figure 1.

The above developments allow us to study the two-particle interaction in a perturbative way. At the tree-level, there are two relevant graphs for the scattering amplitude: the contact-interaction and one-gauge-particle exchange. They are represented in figure 2. The contact-interaction case, already studied in [3], is

$$S^{(0)}_{fi,\lambda} = \mathrm{i}\lambda(2\pi)^4\tilde{\delta}^4(p_1 + p_2 - p'_1 - p'_2).$$

The scattering amplitude for the graph of one-gauge-particle exchange is expressed as

$$S^{(0)}_{fi,\text{exc}} = -\mathrm{i}(2\pi)^5\tilde{\delta}^4(p_1 + p_2 - p'_1 - p'_2)\delta\left(p_1^4 - p_1'^4\right)$$
$$\times\,\Gamma^\mu(p_1 + p'_1)\overline{D}_{\mu\nu}(p_1 - p'_1))\Gamma^\nu(p_2 + p'_2) + [p'_1 \leftrightarrow p'_2]. \tag{27}$$

By noting that $\overline{D}_{i4}$ is the only non-vanishing contribution of $\overline{D}_{\mu\nu}$ in the centre-of-mass frame (where $\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}$, $\mathbf{p}'_1 = -\mathbf{p}'_2 = \mathbf{p}'$ and $|\mathbf{p}| = |\mathbf{p}'| = p$), we get the identity

$$\Gamma^\mu(p_1 + p'_1)\overline{D}_{\mu\nu}(p_1 - p'_1))\Gamma^\nu(p_2 + p'_2) = -\frac{8m\mathrm{i}\beta^2e^2(\mathbf{p}'\times\mathbf{p})}{\kappa(\mathbf{p}-\mathbf{p}')^2}. \tag{28}$$

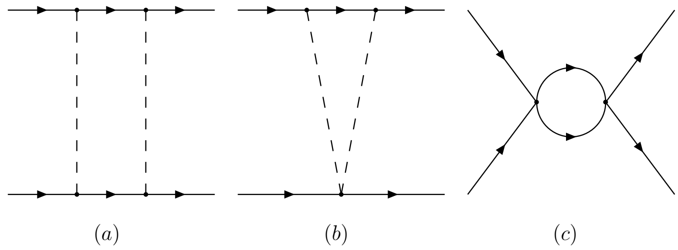

Figure 3. Two-particle scattering at one-loop.

Therefore, if we take into account the combination of possible diagrams, equation (27) reduces to

$$S^{(0)}_{fi,\mathrm{exc}} = -(2\pi)^4\tilde{\delta}^4(p_1+p_2-p_1'-p_2')\left[2\pi\delta\left(p_1^4-p_1'^4\right)\right]\frac{2e^2}{m\kappa}\cot\theta,$$

where θ is the scattering angle. By noting that $M^{(0)}_\lambda = 1$ and $M^{(0)}_{fi,\mathrm{exc}} = 2\pi\delta\big(p_1^4-p_1'^4\big)$, the invariant amplitude for the tree-level scattering is

$$A^{(0)}(p,\theta) = -\lambda - \frac{2\mathrm{i}e^2}{m\kappa}\cot\theta.$$

The diagrams which contribute to the one-loop two-particle scattering amplitude are depicted in figure 3. The bubble diagram ensuing from the contact interaction generates the following scattering amplitude:

$$S^{(1)}_{fi,\lambda} = -\lambda^2(2\pi)^2\tilde{\delta}^4(p_1+p_2-p_1'-p_2')\delta\left(p_1^4+p_2^4-2m\right)\int \mathrm{d}^2k\,\mathrm{d}k^5\,\Delta_F(k)\Delta_F(p_1+p_2-k).$$

From [3], the integration in k^5 can be performed analytically. Then, with $M^{(1)}_\lambda = 2\pi\delta\big(p_1^4+p_2^4-2m\big)$, we find that the invariant amplitude is

$$A^{(1)}_\lambda(p) = \frac{m\lambda^2}{8\pi}\left[\ln\frac{\Lambda}{p^2}+\mathrm{i}\pi\right].$$

Now if we consider the box diagram, the scattering amplitude is

$$\begin{aligned}S^{(1)}_{fi,\mathrm{box}} = {}& -(2\pi)^4\tilde{\delta}^4(p_1+p_2-p_1'-p_2')\delta\left(p_1^4+p_2^4-2m\right)\delta\big(p_1^4-m\big)\delta\big(p_1'^4-m\big)\\ &\times\int \mathrm{d}^4k\,\Gamma^\mu(p_1+k)\overline{D}_{\mu\nu}(k-p_1)\Gamma^\nu(2p_2+p_1-k)\Delta_F(k)\Delta_F(p_1+p_2-k)\\ &\times\Gamma^\alpha(-k+p_1+p_2+p_2')\overline{D}_{\alpha\beta}(k-p_1')\Gamma^\beta(k+p_1)+[p_1'\leftrightarrow p_2'].\end{aligned}$$

If we utilize equation (28) into this equation and integrate over the component k^5, then the invariant amplitude of the box diagram is

$$A^{(1)}_{\mathrm{box}}(\mathbf{p},\mathbf{p}') = -\frac{16m^3\beta^4e^4}{\kappa^2}\int\frac{\mathrm{d}^2k}{(2\pi)^2}\frac{4(\mathbf{k}\times\mathbf{p})(\mathbf{k}\times\mathbf{p}')}{(\mathbf{k}-\mathbf{p})^2(\mathbf{k}-\mathbf{p}')^2(\mathbf{p}^2-\mathbf{k}^2+\mathrm{i}\epsilon)}+[\mathbf{p}'\leftrightarrow-\mathbf{p}']. \tag{29}$$

In this case $M^{(1)}_{\mathrm{box}} = (2\pi)^3\delta\big(p_1^4+p_2^4-2m\big)\delta\big(p_1^4-m\big)\delta\big(p_1'^4-m\big)$. This is just the same expression as in [17] obtained directly by the Schrödinger field theory. If we use Cauchy's theorem for the angular integration, then equation (29) can be rewritten as

$$A^{(1)}_{\mathrm{box}}(p,\theta) = -\frac{e^4}{2\pi m\kappa^2}[\ln|2\sin\theta|+\pi\mathrm{i}].$$

For the triangle diagram in figure 3, the scattering amplitude reads

$$S^{(1)}_{fi,\mathrm{tri}} = \mathrm{i}(2\pi)^3\tilde{\delta}^4(p_1 + p_2 - p'_1 - p'_2)\delta\big(p^4_1 - m\big)\delta\big(p'^4_1 - m\big) \\ \times \int \mathrm{d}^4k\, \Gamma^{\rho\sigma}\Gamma^{\mu}(p_1 + k)\overline{D}_{\mu\rho}(k - p_1)\Gamma^{\nu}(k + p'_1)\overline{D}_{\nu\sigma}(k - p'_1)\Delta_F(k) + [p'_1 \leftrightarrow p'_2].$$

If we use once again the properties of the gauge propagator, together with the centre-of-mass frame and integrate over k^5, then, by identifying $M^{(1)}_{\mathrm{box}} = (2\pi)^2\delta\big(p^4_1 - m\big)\delta\big(p'^4_1 - m\big)$, the invariant amplitude for the triangle diagram is

$$A^{(1)}_{\mathrm{tri}}(\mathbf{p}, \mathbf{p}') = \frac{2\beta e^4}{\kappa^2}\int \frac{\mathrm{d}^2k}{(2\pi)^2}\frac{4(\mathbf{k} - \mathbf{p})\cdot(\mathbf{k} - \mathbf{p}')}{(\mathbf{k} - \mathbf{p})^2(\mathbf{k} - \mathbf{p}')^2} + [\mathbf{p}' \leftrightarrow -\mathbf{p}'].$$

Clearly, this integral may be seen as similar to the expression obtained in [17], simply by performing the shift: $\mathbf{k}' = \mathbf{k} - \mathbf{p}'$. Since the integral is logarithmically divergent, we use the cutoff Λ for the ultraviolet divergences. This allows us to rewrite equation (29) as

$$A^{(1)}_{\mathrm{tri}}(p, \theta) = -\frac{e^4}{2\pi m\kappa^2}\ln\frac{\Lambda^2}{2p^2|\sin\theta|}.$$

Hence, we obtain an expression of the total renormalized invariant amplitude that is in agreement with [17]:

$$A(p, \theta, \mu) = -\lambda_R - \frac{2\mathrm{i}e^2}{m\kappa}\cot\theta + \frac{m}{8\pi}\left[\lambda^2_R - \frac{4e^4}{m^2\kappa^2}\right]\left[\ln\frac{\mu^2}{p^2} + \mathrm{i}\pi\right],$$

where λ_R is the renormalized coupling constant, obtained from the redefinition of λ to eliminate the ultra-violet divergencies, and μ is an arbitrary scale constant. Note that with the choice $\lambda_R = \pm\frac{2e^2}{m|\kappa|}$, the scale dependence vanishes. Moreover, if we choose the upper + sign in λ_R, then we get $A(\theta) = \frac{4\pi\alpha}{m}[\mathrm{i}\cot\theta + \mathrm{sgn}\,\alpha]$, with $\alpha = \frac{e^2}{2\pi m\kappa}$ being the AB parameter. Therefore, this result reproduces the AB amplitude. The Galilean-covariant approach discussed in the introduction has therefore been utilized with path-integral quantization to consider non-relativistic bosons coupled to the CS gauge field. As an application, we have obtained a Lorentz-like (albeit non-relativistic) formulation of the AB effect.

4. Concluding remarks

In this paper, we have provided another illustration of a covariant approach to Galilean field theories based on a (4, 1) Minkowski space: the construction of non-relativistic models of complex scalar bosons coupled to Chern–Simons gauge fields. The group-theoretical motivation is that the 11-dimensional central extension of the Galilei algebra of (3, 1) space-time is a Lie subalgebra of the inhomogeneous Lorentz algebra of the (4, 1) Minkowski space. We have built various models by starting with the Carroll–Field–Jackiw three-dimensional Chern–Simons term, for which the usual Lorentz covariance is broken. As an application of this covariant formalism, we have examined the Aharonov–Bohm effect and retrieved the invariant scattering amplitude up to one-loop already found in the literature. Further study of applications of this formulation of Galilei-invariant field theories in condensed matter physics and to many-body problems is under way.

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References

[1] Takahashi Y 1988 *Fortschr. Phys.* **36** 63
Takahashi Y 1988 *Fortschr. Phys.* **36** 83
[2] Omote M, Kamefuchi S, Takahashi T and Ohnuki Y 1989 *Fortschr. Phys.* **37** 933
Takahashi Y 1987 *Wandering in the Fields: Festschrift for Professor Kazahiko Nishijima on the Occasion of his Sixtieth Birthday* ed K Kwarabayashi and A Ukawa (Singapore: World Scientific) p 117
[3] Abreu L M, de Montigny M, Khanna F C and Santana A E 2003 *Ann. Phys., NY* **308** 244
[4] de Montigny M, Khanna F C, Santana A E, Santos E S and Vianna J D M 1999 *Ann. Phys., NY* **277** 144
[5] Santos E S, de Montigny M, Khanna F C and Santana A E 2004 *J. Phys. A: Math. Gen.* **37** 9771
[6] de Montigny M, Khanna F C and Santana A E 2003 *J. Phys. A: Math. Gen.* **36** 2009
[7] de Montigny M, Khanna F C, Santana A E and Santos E S 2001 *J. Phys. A: Math. Gen.* **34** 8901
[8] de Montigny M, Khanna F C and Santana A E 2003 *Int. J. Theor. Phys.* **42** 649
[9] Chern S S and Simons J 1974 *Ann. Math.* **99** 48
[10] Siegel W 1979 *Nucl. Phys.* B **156** 135
Jackiw R and Templeton S 1981 *Phys. Rev.* D **23** 2291
Schonfeld J 1981 *Nucl. Phys.* B **185** 157
[11] Deser S 1998 Chern–Simons terms as an example of the relation between mathematics and physics *Preprint* math-ph/9805020
[12] Dunne G 1995 *Self-Dual Chern–Simons Theories* (New York: Springer)
Dunne G 2000 *Topological Aspects of Low Dimensional Systems (Les Houches, NATO Advanced Study Institute* vol 69*)* ed A Comtet, T Jolicoeur, S Ouvry and F David (Berlin: Springer) p 176 (*Preprint* hep-th/9902115)
[13] Fradkin E 1991 *Field Theories of Condensed Matter Systems* (New York: Addison-Wesley) chapters 7–8
[14] Jackiw R and Pi S Y 1990 *Phys. Rev. Lett.* **64** 2969
Jackiw R and Pi S Y 1990 *Phys. Rev.* D **42** 3500
[15] Aharonov Y and Bohm D 1959 *Phys. Rev.* **115** 485
Ruijsenaars S N M 1983 *Ann. Phys., NY* **146** 1
[16] Hagen C R 1985 *Phys. Rev.* D **31** 848
[17] Bergman O and Lozano G 1994 *Ann. Phys., NY* **229** 416
[18] Boz M, Fainberg V and Pak N K 1995 *Phys. Lett.* A **207** 1
Boz M, Fainberg V and Pak N K 1996 *Ann. Phys., NY* **246** 347
[19] Gomes M, Malbouisson J M C and da Silva A J 1997 *Phys. Lett.* A **236** 373
Gomes M, Malbouisson J M C and da Silva A J 1998 *Int. J. Mod. Phys.* A **13** 3157
[20] Fushchich W I and Nikitin A G 1994 Symmetries of Equations in Quantum Mechanics (New York: Allerton Press) section 26
Fushchich W I and Nikitin A G 1980 *J. Phys. A: Math. Gen.* **13** 2319
[21] de Azcárraga J A and Izquierdo J M 1995 *Lie Groups, Lie Algebras, Cohomology and Some Applications in Physics* (Cambridge: Cambridge University Press)
[22] Pinski G 1968 *J. Math. Phys.* **9** 1927
Soper D E 1976 *Classical Field Theory* (New York: Wiley) section 7.3
Duval C, Burdet G, Künzle H P and Perrin M 1985 *Phys. Rev.* D **31** 1841
Künzle H P and Duval C 1994 *Semantical Aspects of Spacetime Theories* ed U Majer and H J Schlt (Mannheim: BI-Wissenschaftsverlag) p 113
Kapuścik E 1986 *Acta Phys. Pol.* B **17** 569
Kapuścik E 1980 *Nuovo Cimento* A **58** 113
Kapuścik E 1981 *Acta Phys. Pol.* B **12** 81
Abreu L M, Santana A E and Ribeiro Filho A 2002 *Ann. Phys., NY* **297** 396
[23] Hassaïne M and Horváthy P A 2000 *Ann. Phys., NY* **282** 218
Hassaïne M and Horváthy P A 2001 *Lett. Math. Phys.* A **57** 33
[24] Jackiw R 2002 *Lectures on Fluid Dynamics—A Particle Theorist's View of Supersymmetric, Non-Abelian, Noncommutative Fluid Mechanics and d-Branes* (Berlin: Springer) (*Preprint* physics/0010042)
Bazeia D and Jackiw R 1998 *Ann. Phys., NY* **270** 246
Jackiw R and Polychronakos A P 1999 *Commun. Math. Phys.* **207** 107
Jackiw R and Polychronakos A P 1999 *Proc. Steklov Inst. Math.* **226** 193 (*Preprint* hep-th/9809123)
[25] Lee C, Lee K and Min H 1990 *Phys. Lett.* B **252** 79
[26] Dunne G and Trugenberger C 1991 *Phys. Rev.* D **43** 1323
[27] Fainberg V Ya, Pak N K and Shikakhwa M S 1997 *J. Phys. A: Math. Gen.* **30** 3947
Fainberg V Ya and Shikakhwa M S 1996 *Phys. Rev.* D **53** 5765
[28] Dimock J 1977 *Commun. Math. Phys.* **57** 51

[29] Manuel C and Tarrach R 1991 *Phys. Lett.* B **268** 222
[30] Bég M A B and Furlong R C 1985 *Phys. Rev.* D **31** 1370
[31] Carroll S M, Field G B and Jackiw R 1990 *Phys. Rev.* D **41** 1231
Jackiw R and Pi S Y 2003 *Phys. Rev.* D **68** 104012
Jackiw R 2004 *Proc. 3rd Int. Symp. Quantum Theory and Symmetries* ed P C Argyres, T J Hodge, F Mansouri, J J Scanio, P Suranyi and L C R Wijewardhana (Singapore: World Scientific) (*Preprint* gr-qc/0310115)
[32] Le Bellac M and Lévy-Leblond J M 1973 *Nuovo Cimento* B **14** 217
[33] Rousseaux G 2005 *Europhys. Lett.* **71** 15
[34] Lee K 1991 *Phys. Lett.* B **255** 381
Lee K 1991 *Phys. Rev. Lett.* **66** 553
[35] Dunne G, Jackiw R, Pi S Y and Trugenberger C 1991 *Phys. Rev.* D **43** 1332
Dunne G, Jackiw R, Pi S Y and Trugenberger C 1992 *Phys. Rev.* D **45** 3012 (erratum)
[36] Bak D and Bergman O 1995 *Phys. Rev.* D **51** 1994